The Effectiveness of Multiobjective Optimizer in Single-objective Optimization Environment

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ABSTRACT

This paper presents two new approaches for transforming a single-objective problem into a multi-objective problem. The first approach is based on relaxation of the constraints of the problem and the other is based on the addition of noise to the objective value or decision variable. Intuitively, these approaches provide more freedom to explore and a reduced likelihood of becoming trapped in local optima.

Through numerical examples, we showed that the multi-objective versions produced by relaxing constraints can provide good results and that using the addition of noise can obtain better solutions when the function is multimodal and separable.

Categories and Subject Descriptors

G.1.6 [**Global optimization**]: Multi-objective optimization—*multi-objectivizing*

General Terms

ALGORITHMS

Keywords

Multiobjectivizing, Genetic Algorithms, Multi-objective optimization

1. DESCRIPTION OF THE NEW MULTI-OBJECTIVIZATION APPROACH

In this paper, we propose two "new multi-objectivization" [1] approaches based on the addition of new objectives as follows:

- Relaxing the constraints of the problem.
- Adding noise to the objective value or decision variables.

The multi-objectivization approach translates single-objective optimization problems (SOOP) into multi-objective optimization problems (MOOP) and then applies EMO to the translated problem. The aims of these approaches are to increase paths to the global optimum that are difficult to obtain under the original SOOP and to maintain the diversity of the population.

These approaches have a low risk of providing solutions far from the optimal solutions as these approaches always deal with the original SOOP objective. In addition, these approaches can treat many types of problems and hardly produce new tasks such as decomposition of a problem into sub-problems.

The first approach is based on the concept of constraint relaxation. In this approach, a trade-off between the original and the relaxed objectives can be brought by differences in two constrains. Therefore, a search of EMO can be concentrated around the constrains of a problem. This approach can be expected to search effectively for the global optimum along the boundary of the feasible regions settled by the original constraints.

The another approach takes advantage of escaping from local optima. In this approach, the trade-off relation is introduced by the difference between the original objective and the new objective with noise. This approach will be useful for escaping from local optima using trade-off regions.

2. NUMERICAL EXAMPLES

In this paper, we describe application of the proposed approaches to two types of numerical experiment : the 0/1 knapsack problem with 750 items and typical test functions (Rastrigin, Schwefel, etc.) . In implementing our proposed approach, we used two types of GA:

- "ga2k"[2] [2] as a single-objective GA.
- "NSGA-II"[3] as a multi-objective GA.

2.1 Implementation of GA

In these experiments, GAs applied to the two types of experiment used bit coding. Similarly, two-point crossover and bit flip were implemented as for crossover and mutation. We performed 30 trials and all results are shown as averages of 30 trials. In addition, the terminal condition of all experiments was 200 generations.

[1]This term was used previously by Knowles et al.[1].

[2]This algorithm is based on the island GA model. A prototype implementation has been written in C++ and can be downloaded from [2].

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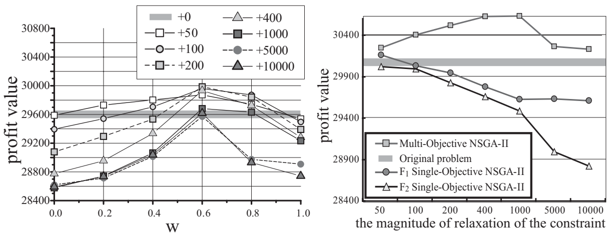


(a) The results of ga2k (b) The results of NSGA-II

Figure 1: The results of knapsack problem.

2.2 Multi-objectivization with relaxation of constraints

We used the 0/1 knapsack problem with 750 items and multi-objectivized the problem. This multi-objectivization problem was a maximization problem.

As many codings lead to infeasible solutions, we should implement a repair method as a constraint handling technique. We used the repair method proposed by Zitler et al.[4]. We used the repair method when the codings violated the relaxed capacity. In addition, we used $\alpha = 3.0$ based on the results of our pilot study.

2.2.1 Results

The results are shown in Fig. 1. The horizontal axis in Fig. 1 (a) indicates the value of the weight parameter ω and that in Fig. 1 (b) indicates the magnitude of relaxation of the constraints settled by the difference between the original capacity and relaxed . Fig. 1 (b) shows the results of 4 experiments based on the implementation of objectives (f_1 and f_2).

The grey bands in Fig. 1 indicate the results of the original SOOP obtained by ga2k and NSGA-II. Therefore, by investigating whether the results of multi-objectivization were higher or lower than the grey band, it was possible to determine the useful of multi-objectivization for this problem.

The experimental results confirmed that this multi - objectivization approach using a multi-objective GA is effective for the 0/1 knapsack problem. In addition, we found that there is an optimum magnitude of relaxation for an additional objective.

2.3 Multi-objectivization with addition of noise

We examined the effectiveness of multi-objectivization by adding noise according using typical test functions (Rastrigin, Schwefel, etc.). In these experiments, all problems were minimization problems.

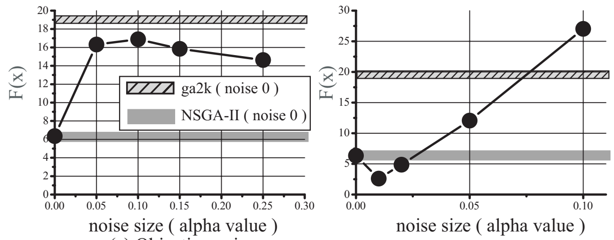
2.3.1 Test functions

In this example, we used functions from the perspective of the modality (unimodal or multimodal) of function and epistasis among decision variables (separable or non-separable). We used 5 types of function: Rastrigin, Schwefel, Ridge, Rotated Rastrigin and Rotated Schwefel functions. We treated all functions as having 10 decision variables in this example.

In this example, the grey coding is used and the length of the chromosome is 20 bits per decision variable. The simulation of all functions is terminated when the generation reaches over 200.

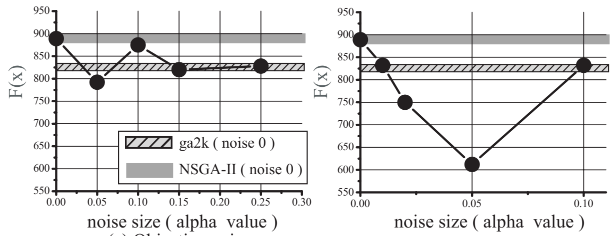
2.3.2 Results

Because space is very limited, the performances of 2 functions are shown in Fig. 2 and 3.



(a) Objective noise (b) Variable noise

Figure 2: The result of Ridge



(a) Objective noise (b) Variable noise

Figure 3: The result of Rotated Schwefel

In these figures, the horizontal axes show the values of α, which is the parameter for adjusting the magnitude of the noise, and the vertical axes show the values of the original functions. The left (a) and right (b) figures show the results of adding noise to the objective value and to the decision variables, respectively. In addition, the grey bands in the figures indicate the results of the original function value (i.e., the magnitude of noise is zero) obtained by ga2k and NSGA-II, the bands marked with diagonal lines show the results of ga2k and the other bands show those of NSGA-II.

From the above results, it is apparent that the multi-objectivization approach using the addition of noise to decision variables is very effective for multimodal functions with epistasis, such as the Rotated Rastrigin and Rotated Schwefel functions.

3. REFERENCES

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